ELECTRIC SIMULATION OF NONLINEAR PROBLEMS IN HEAT AND MASS TRANSFER

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A method is described for the electric simulation of nonlinear heat- and mass-transfer problems on two resistance networks. The temperature is simulated on one network and the mass-transfer potential on the other. The mass-transfer potential has a discontinuity of the first kind because of a moving zone boundary on which intense phase transitions occur.

1. In the electric simulation of the nonlinear equations of heat and mass transfer

$$c\gamma \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial t}{\partial x}\right) + \varepsilon r \frac{\partial u}{\partial \tau},$$
$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left(a_m \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x}\left(a_m \delta \frac{\partial t}{\partial x}\right), \tag{1}$$

where the coefficients λ, c, a_m, δ, . . . depend on u and t [1], Eqs. (1) can be written in finite-difference form

$$c\gamma \frac{t_0 - t_{0-\Delta\tau}}{\Delta\tau} = \lambda_1 \frac{t_1 - t_0}{\Delta x^2} + \lambda_2 \frac{t_2 - t_0}{\Delta x^2} + \varepsilon r \frac{u_0 - u_{0-\Delta\tau}}{\Delta\tau},$$
$$\frac{u_0 - u_{0-\Delta\tau}}{\Delta\tau} = a_{m_1} \frac{u_1 - u_0}{\Delta x^2} + a_{m_2} \frac{u_2 - u_0}{\Delta x^2} \tag{2}$$
$$+ a_{m_1}\delta_1 \frac{t_1 - t_0}{\Delta x^2} + a_{m_2}\delta_2 \frac{t_2 - t_0}{\Delta x^2}.$$

A method for the electric simulation of such a system of nonlinear equations was presented by L. A. Kozdova in [2] and in more general form in [3]. The continuous-medium electric simulator consists of two networks of ohmic resistors (Fig. 1). In network *a* the electric potential V simulates the temperature t ($t = N_t V$); in network b the electric potential W is proportional to the moisture content u = $N_u W$. The heat flux j_q and the mass flux j_m are proportional to the network currents

$$j_q = N_{j_q} i_1; \quad j_m = N_{j_m} i_2.$$

The difference equations (2) are simulated on the networks by relations obtained from Kirchhoff's law:

$$\frac{V_0 - V_{0-\Delta\tau}}{R_\tau^t} = \frac{V_1 - V_0}{R_1^t} + \frac{V_2 - V_0}{R_2^t} + \frac{V_3 - V_0}{R_u^t}, \tag{3}$$

$$\frac{W_0 - W_{0-\Delta\tau}}{R_\tau^u} = \frac{W_1 - W_0}{R_1^u} + \frac{W_2 - W_0}{R_2^u} + \frac{W_3 - W_0}{R_t^u}. \tag{4}$$

2. These network simulators can also be used to solve more complicated problems such as the simulation of the drying of a porous medium with a moving surface of intense evaporation.

We consider a one-dimensional problem where the position of the surface of intense evaporation is determined by a single coordinate $\eta(\tau)$. The velocity of the surface is $\dot{\eta} = d\eta/dt$. The thermophysical

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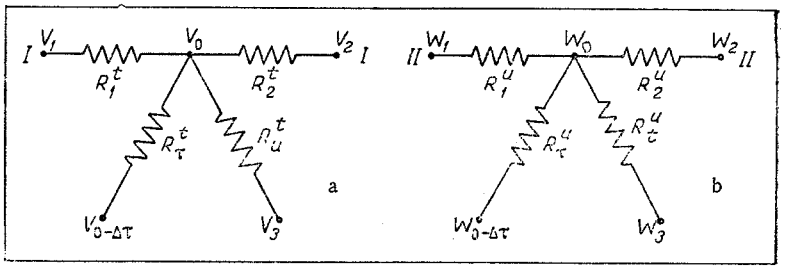

Fig. 1. Circuit diagrams of separate sections simulating a one-dimensional heat- and mass-transfer problem. I–I simulates t and II–II simulates u.

characteristics λ, c, λ_m, c_m, δ, and ε depend on both t and u in general. They are discontinuous at the boundary $x = \eta$. Relatively simple methods of electric simulation taking account of the propagation of a phase transition front are given in the literature [4] for the case of a single unknown function and constant thermophysical parameters. Since there is a large variation of thermophysical characteristics during the drying process the use of such constant parameter models gives a very crude approximation.

We present an electric simulator for solving the set of differential equations (1) for two functions u and t with a moving surface of intense evaporation, i.e., with a discontinuity in the function u at $x = \eta$.

We first find the velocity $\dot{\eta}$ at time τ for known distribution laws t(x) and u(x) at that instant. To this end we consider a thin layer with coordinates $\eta - 0$ and $\eta + 0$ in the vicinity of the moving surface where the intense phase transitions occur. We determine the amount of heat entering the elementary layer per unit time [5]. The heat fluxes entering the layer due both to the nonuniform temperature distribution and to the motion of the layer through a medium with a different heat content are

$$(-\lambda\nabla t - c\gamma\dot{\eta}t)_{\eta-0} - (-\lambda\nabla t - c\gamma\dot{\eta}t)_{\eta+0}. \tag{5}$$

Similarly the total amount of moisture entering the thin layer per unit time is

$$(-\lambda_m\nabla u - \lambda_m\delta\nabla t - \dot{\eta}c_m u)_{\eta-0} - (-\lambda_m\nabla u - \lambda_m\delta\nabla t - \dot{\eta}c_m u)_{\eta+0}. \tag{6}$$

We denote by E the criterion of a phase transition, the ratio of the weight of liquid evaporated in the layer under consideration to the total weight of liquid entering the layer ($0 < E < 1$). The porous medium contains capillaries of various diameters. It is assumed that the moisture in the largest capillaries evaporates instantly as the boiling surface passes. The magnitude of E depends in general on the parameters t, u, ∇t, and ∇u at $x = \eta$ before and after the passage of the evaporation surface. The equation for $\dot{\eta}$ can be derived from the law of conservation of energy

$$\frac{d\eta}{dt} = \frac{[\lambda\nabla t - rE(\lambda_m\nabla u + \lambda_m\delta\nabla t)]_{\eta-0} - [\lambda\nabla t - rE(\lambda_m\nabla u + \lambda_m\delta\nabla t)]_{\eta+0}}{(-c\gamma t + rEc_m u)_{\eta-0} - (-c\gamma t + rEc_m u)_{\eta+0}}. \tag{7}$$

If the thermophysical characteristics λ, λ_m, c, c_m, δ, and E and the parameters t, ∇t, u and ∇u are known in front of and behind the evaporation surface the velocity of the surface can be calculated from Eq. (7).

The temperature of the medium is a continuous function of x and τ, but at the point $x = \eta$ the partial derivatives of this function have finite discontinuities; the moisture content has a discontinuity of the first kind at the evaporation surface.

The equations of heat and mass transfer are simulated on two resistance networks. The first of Eqs. (1) takes the form

$$c\gamma\frac{\partial t}{\partial\tau} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial t}{\partial x}\right) + \varepsilon r\frac{\partial u}{\partial\tau} - \bar{\delta}(x - \tau\dot{\eta})\,Q, \tag{8}$$

where $\bar{\delta}$ is the Dirac delta function; Q, the amount of heat expended in evaporation per unit time in the layer under consideration, is determined by Eq. (5). We construct a resistance network to simulate t (Fig. 2a). We assume that at time τ_0 the values of Δx and $\Delta\tau$ are chosen in such a way that $\Delta x/\Delta\tau$ is equal

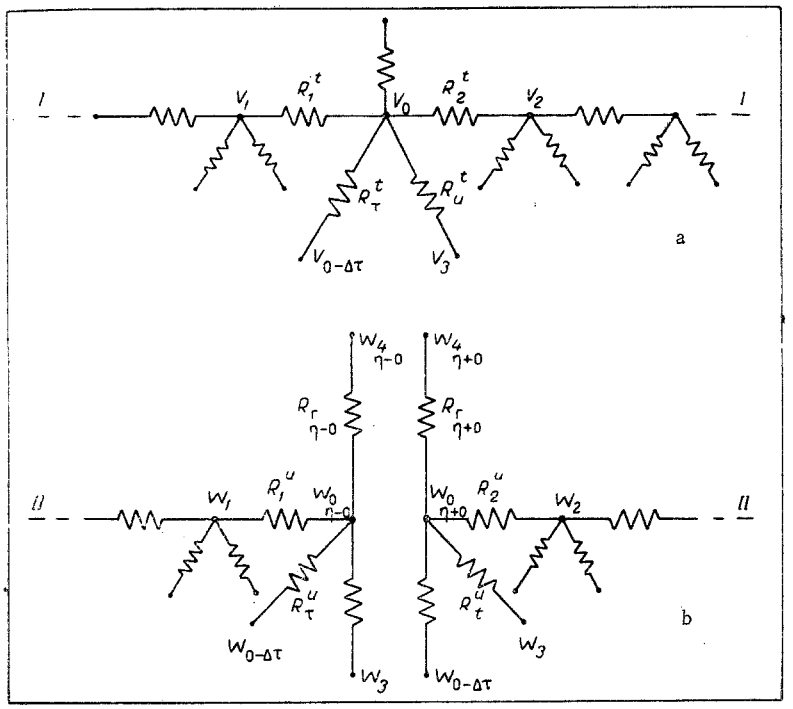

Fig. 2. Electric circuits for simulating a one-dimensional heat- and mass-transfer problem with a moving surface of intense phase transition.

to the velocity of the evaporation surface. This implies that in one step on the simulator the surface of discontinuity $x = \eta$ is displaced by one cell of the network.

We note that Δx and $\Delta\tau$ can be considered constants for a small change in $\dot{\eta}$. In general, after each step it is necessary to determine η and $\Delta\tau - \dot{\eta}\Delta x$ and to change the resistance R_τ^t and R_τ^u by amounts proportional to $\Delta\tau$. For a small $\Delta\tau$ the position of the discontinuity can be displaced a large number of cells.

Suppose the surface is in the position corresponding to the terminal of the network I–I with potential V_0. In addition to the currents $(V_0 - V_0 - \Delta\tau)/R_\tau^t$ and V_3/R_u^t simulating the first terms in Eqs. (8) it is necessary to remove a current from the terminal at potential V_0 to simulate the loss of heat during the time $\Delta\tau$ due to evaporation in the moving layer during its displacement by Δx. This amount of heat can be calculated by knowing the parameters u, t, ∇u, and ∇t in the neighborhood of the discontinuity, i.e. by determining them beforehand with the second network. The circuit diagram of network II–II simulating u is shown in Fig. 2b. Since the function u is discontinuous at $x = \eta$, this must correspond to a physical discontinuity in the electric network, and at the position of the discontinuity we must obtain not one potential but two, $W_{0_{\eta-0}}$ and $W_{0_{\eta+0}}$. Each of the half networks ($x < \eta$; $x > \eta$) represents an independent boundary-value problem. Since the layer we constructed where evaporation occurs ($\eta - 0$, $\eta + 0$) has been removed, and the terminals at potential $W_{0_{\eta-0}}$ and $W_{0_{\eta+0}}$ correspond to the boundary points of this layer, it is not necessary to assign special points to simulate boiling. However, the boundary conditions at ($\eta - 0$) and ($\eta + 0$) must be satisfied. This requires supplying currents proportional to the boundary heat fluxes to the terminals at potentials $W_{0_{\eta-0}}$ and $W_{0_{\eta+0}}$ by connecting them to a high potential W_4 of the potentiometer through the high resistances $R_{r_{\eta-0}}$ and $R_{r_{\eta+0}}$:

$$i_{r_{\eta-0}} = \frac{W_{4\,\eta-0}}{R_{r_{\eta-0}}} = \frac{1}{N l_m}\left(-\lambda_m \frac{u_{0-\Delta\tau} - u_{1-\Delta\tau}}{\Delta x} - \lambda_m \delta \frac{t_{0-\Delta\tau} - t_{1-\Delta\tau}}{\Delta x}\right), \tag{9}$$

just as is ordinarily done in simulating conditions of the second kind; $i_{r_{\eta+0}}$ is determined similarly.

The simulation of a problem with a moving front on which the potential experiences a discontinuity of the first kind requires a discontinuity in the network since the electric potential W varies continuously in a continuous resistance network for any current sources.

On p. 241 of [1] are shown changes in temperature and moisture content of a typical porous body (a ceramic plate 51 mm thick) when dried from both sides by infrared radiation. Periods of drying are clearly visible on the temperature curves: an initial period of heating, a period of constant rate of drying during which the temperature is practically constant, and the period we are interested in beyond the critical point K_2 where sharp bends in the curves $t = f(\tau)$ for $x = \text{const}$ are clearly visible. These bends occur at the instants the surface of intense evaporation passes through the given point. Since the derivatives $\partial t/\partial\tau$, $\partial t/\partial x$. . . experience discontinuities of the first kind, Eqs. (1) are valid outside the surface of intense evaporation. We have performed the electric simulation of transfer processes during an interval $\tau = 360$–534 min for the experiment indicated above. The thermophysical parameters were selected as in [6]. At the initial time $\tau = 360$ min the temperature was assumed constant, u(x) was considered distributed parabolically, and the thermophysical coefficients c, λ, and a_m were assumed linear functions of u; $\delta = \text{const}$. The boundary conditions were simulated as described in [3].

The function $t = f(\tau, x)$ obtained by electric simulation is in satisfactory agreement with the actual experiment.

3. The solution of nonlinear heat- and mass-transfer problems on ohmic resistance networks is a laborious and relatively prolonged operation, and therefore it is desirable to first obtain an approximate solution by replacing the variable thermophysical characteristics λ, c, a_m etc. by some average constant values and simulating the process on an RC network [7, 8]. If a front of intense phase transition moves in the medium, we transform to a moving coordinate system with origin at $x = \eta$ and write (1) in dimensionless coordinates for $x < \eta$ [9]

$$-\frac{\partial T}{\partial X}\dot{\eta}+\frac{\partial T}{\partial \mathrm{Fo}}=\frac{\partial^2 T}{\partial X^2}+\mathrm{Ko}^*\left(-\dot{\eta}\,\frac{\partial\Theta}{\partial X}+\frac{\partial\Theta}{\partial \mathrm{Fo}}\right),$$
$$-\frac{\partial\Theta}{\partial X}\dot{\eta}+\frac{\partial\Theta}{\partial \mathrm{Fo}}=\mathrm{Lu}\,\frac{\partial^2\Theta}{\partial X^2}+\mathrm{Lu\,Pn}\,\frac{\partial^2 T}{\partial X^2}. \qquad (10)$$

In (10) the coordinate X is given in the moving system; $\dot{\eta}$ is dimensionless.

We consider the special case when T and Θ depend only on X but not on Fo in the moving coordinate system. Then (10) takes the form

$$\mathrm{Lu}\,\frac{d^2T}{dX^2}+\dot{\eta}(\mathrm{Lu}+1)\frac{dT}{dX}+\dot{\eta}(\mathrm{Lu\,Pn\,Ko}^*+\dot{\eta})T=\text{const},$$
$$\mathrm{Lu}\,\frac{d^2\Theta}{dX^2}+\dot{\eta}(\mathrm{Lu}+1)\frac{d\Theta}{dX}+\dot{\eta}(\mathrm{Lu\,Pn\,Ko}^*+\dot{\eta})\Theta=\text{const},$$

i.e. T and Θ satisfy the same ordinary differential equation. The roots of its characteristic equation are: $k_3 = 0$

$$k_{1,2}=-\frac{\dot{\eta}(\mathrm{Lu}+1)^2\pm\sqrt{(\mathrm{Lu}-1)^2\dot{\eta}^2-4\,\mathrm{Lu}^2\,\mathrm{Pn\,Ko}^*\dot{\eta}}}{2\,\mathrm{Lu}},$$

and the general solution has the form

$$T=A_1\exp(k_1x)+A_2\exp(k_2x)+A_3,$$
$$\Theta=B_1\exp(k_1x)+B_2\exp(k_2x)+B_3.$$

A similar solution can be obtained for $X > 0$. The velocity can be determined by the approximate formula

$$\frac{d\eta}{d\,\mathrm{Fo}}\simeq-\frac{\frac{\partial T}{\partial X}(1-E\,\mathrm{Ko\,Pn\,Lu})}{T\left(\frac{c_{\eta+0}}{c_{\eta-0}}-1\right)-\mathrm{Ko}\,E\left(\frac{c_m}{c_{\eta+0}}\Theta_{\eta+0}-\Theta_{\eta-0}\right)},$$

where Ko, Pn, Lu, and $\partial T/\partial X$ are given at $X = -0$.

NOTATION

τ is the time;
t is the temperature;
u is the moisture content;
λ is the thermal conductivity;
$c\gamma$ is the volumetric heat capacity;
a_m is the mass-transfer potential conductivity;
ε is the ratio of the change of mass due to phase transformation to the total change of mass;
r is the latent heat of phase change;
δ is the thermogradient coefficient;
$\bar{j}_g$ is the heat flux vector;
$\bar{j}_m$ is the mass flux vector;
R^t is the resistance carrying the current which simulates heat flux;
R^u is the resistance carrying the current which simulates flow of moisture;
V is the potential simulating temperature;
W is the potential simulating mass-transfer potential or moisture content;
T is the dimensionless temperature;
Θ is the mass-transfer potential;
Fo is the Fourier number;
Lu is the Lykov number;
Ko is the Kossovich number;
Pn is the Posnov number;
E is the relative change of mass due to phase transitions in the moving region of intense phase transition.

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